Solving punctured multi black hole initial data with finite element method

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- Polynomial order effect
- Comparison to spectral method
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Puncture type initial data in NR

$$R + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j K^j{}_i - \nabla_i K = 0$$

Vacuum spacetime
Involve BH only

$$\gamma_{ij} = \psi^4 f_{ij},$$
$$K_{ij} = \psi^{-2}\hat{K}_{ij},$$

$$K = 0$$

$$\hat{K}_{ij} = \frac{3}{2}\sum_I \frac{1}{r_I^2}[2P^I_{(i}n^I_{j)} - (f_{ij} - n^I_i n^I_j)P^k_I n^I_k$$
$$+ \frac{4}{r_I} n^I_{(i}\epsilon_{j)k\ell} S^k_I n^\ell_I],$$

$$-(\partial_x^2 + \partial_y^2 + \partial_z^2)\psi = \frac{1}{8}\hat{K}^{ij}\hat{K}_{ij}\psi^{-7}$$

Approach to the black holes ($r_I \to 0$), ψ will diverge

$$\psi \equiv 1 + \sum_I \frac{m_I}{2r_I} + u,$$

$$-(\partial_x^2 + \partial_y^2 + \partial_z^2)u = \frac{1}{8}\hat{K}^{ij}\hat{K}_{ij}\psi^{-7}$$

u is C^2 at $r_I = 0$

[Brandt, Bruegman, PRL (1997)]

$$u \to 0 \text{ when } r \to \infty.$$

approximate Dirichlet boundary condition

$$u = d(x) \text{ at } \partial\Omega,$$

with $d = 0$

approximate Robin boundary condition

$$\vec{n} \cdot \nabla u + \alpha(x) u = b(x) \text{ at } \partial\Omega,$$

with $b = 0$ and $\alpha = \frac{1}{r}\frac{\partial r}{\partial n}$

Based on the assumption $u = \dfrac{a}{r}$, a corresponds to the monopole of the spacetime (mass)

Weak form

$$-\nabla^2 u = f(u) \text{ in } \Omega,$$

$$\begin{aligned} & \int_\Omega (\nabla u)\cdot(\nabla v) d^3x + \int_{\partial\Omega} \alpha u v ds \\ = & \int_\Omega f(u) v d^3x + \int_{\partial\Omega} b v ds. \end{aligned}$$

$$u = u^i \phi_i$$

$$\begin{aligned} & u^i \left[\int_\Omega (\nabla \phi_i)\cdot(\nabla \phi_j) d^3x + \int_{\partial\Omega} \alpha \phi_i \phi_j ds \right] \\ = & \int_\Omega f(u) \phi_j d^3x + \int_{\partial\Omega} b \phi_j ds. \end{aligned}$$

Use Newtonian iteration method to solve above non-linear system

$$u_{(n+1)} = u_{(n)} + \Delta u^i \phi_i$$

$$\left[F'_{ij} - M_{ij} - \alpha_{ij}\right] \Delta u^j = \int_\Omega (\nabla u_{(n)}) \cdot (\nabla \phi_i) d^3x$$
$$+ \int_{\partial\Omega} \alpha u_{(n)} \phi_i ds - \int_\Omega f(u_{(n)}) \phi_i d^3x - \int_{\partial\Omega} b\phi_j ds$$

$$M_{ij} = \int_\Omega (\nabla \phi_i) \cdot (\nabla \phi_j) d^3x,$$
$$F'_{ij} = \int_\Omega \frac{df}{du}(u_{(n)}) \phi_i \phi_j d^3x,$$
$$\alpha_{ij} = \int_{\partial\Omega} \alpha \phi_i \phi_j ds.$$

PHG and CaPHG

http://lsec.cc.ac.cn/phg/download.htm

PHG 三维自适应有限元软件平台

Parallel Hierarchical Grid

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Related Link

PHG 源码：phg-0.9.2-20150107.tar.bz2

相关下载:

所有版本 (all distributions)>>

集成电路寄生参数提取软件 ParAFEMImp
ParAFEMImp is a parallel package for wideband impedence extraction in very complicated geometries of conductor. It is based on the recently developed adaptive finite element method for the circuit/field couplings problems and the development of PHG. ParAFEMImp has the potential of good parallel scalbility.

三维并行结构分析软件 PHG-Solid
PHG-Solid is an open source parallel adaptive FEM software for 3D structural analysis. It is based on the 3D parallel adaptive finite element toolbox PHG. It features parallel adaptive finite element analysis for pure 3D structures.

CaPHG
The goal of CaPHG is to integrate Cactus with PHG, an FEM library to help the application development with both FEM and DG methods on unstructure meshes. The motivation is to take advantage of the programmability of the Cactus computational framework and the performance and scalability of the PHG library to provide an integrated problem solving environment to solve large scale scientific problems on high-end computing facilities.

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PHG and CaPHG

https://www.cct.lsu.edu/~jtao/cct/CaPHG/CaPHG.html

Cactus on Unstructured Meshes with CaPHG

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(last updated: 07/04/2014)

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Cactus Parallel Hierarchical Grid
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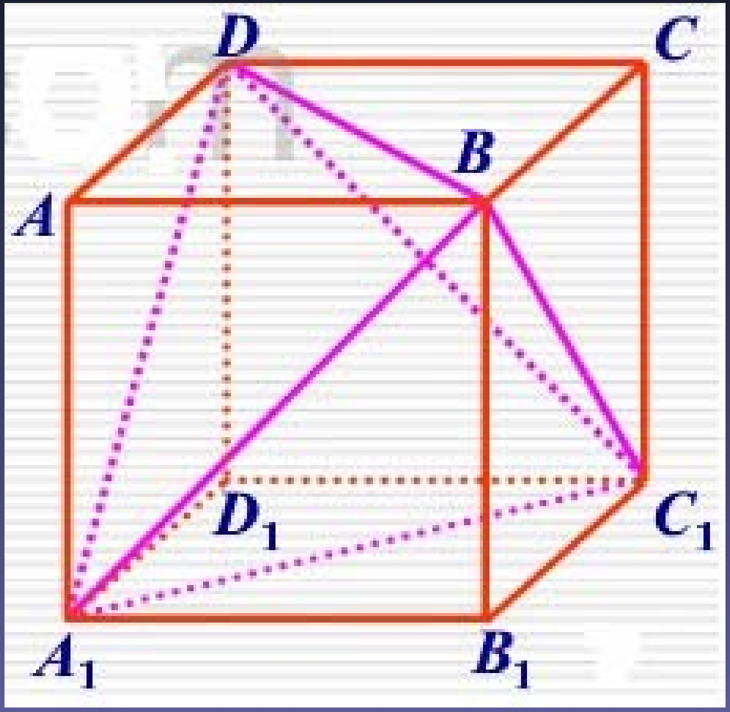
Initial mesh grid generating 1

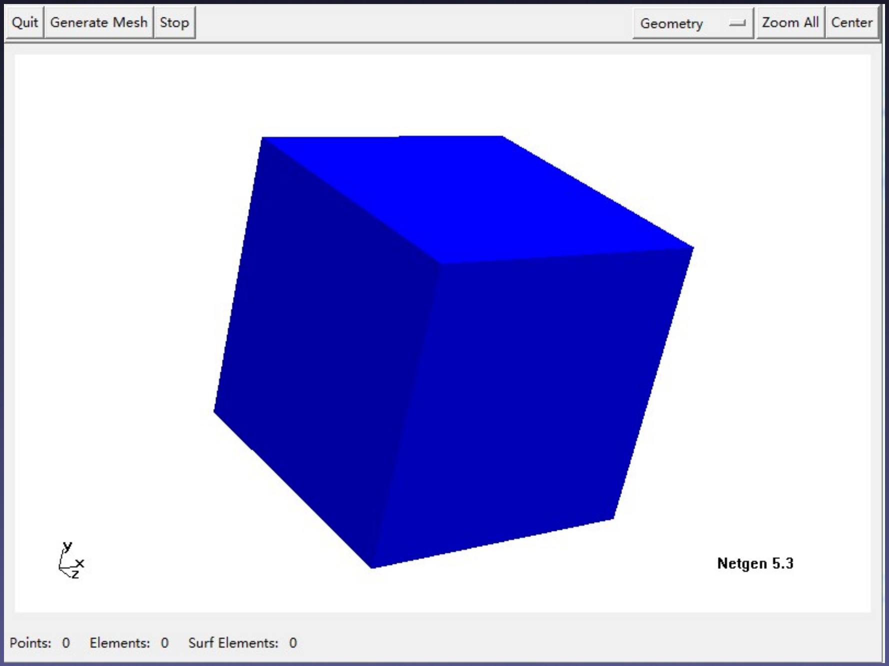
Simplest decomposition of a box:

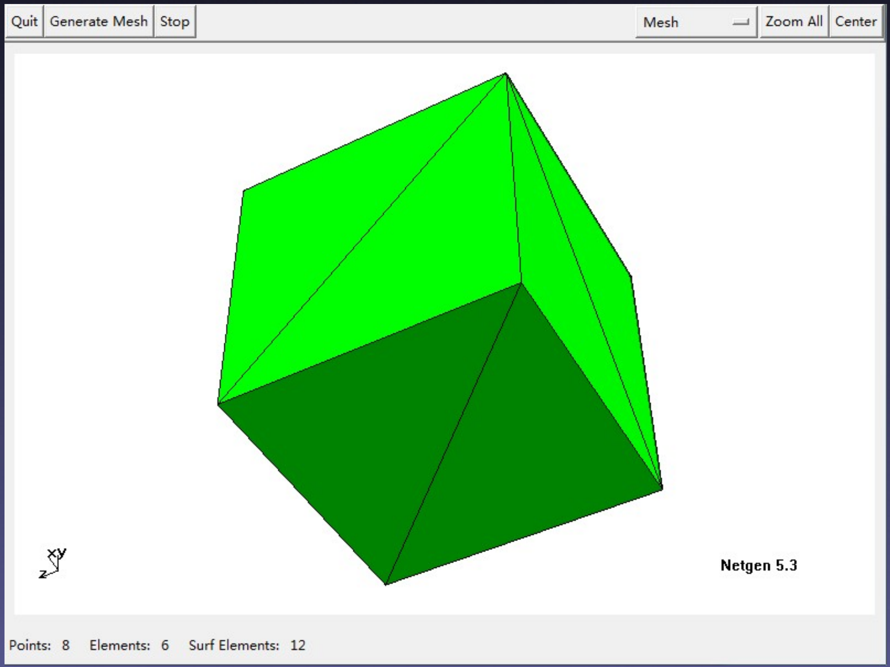
8 vertices and 5 elements



Initial mesh grid generating 2

Using NetGen to generate mesh grid

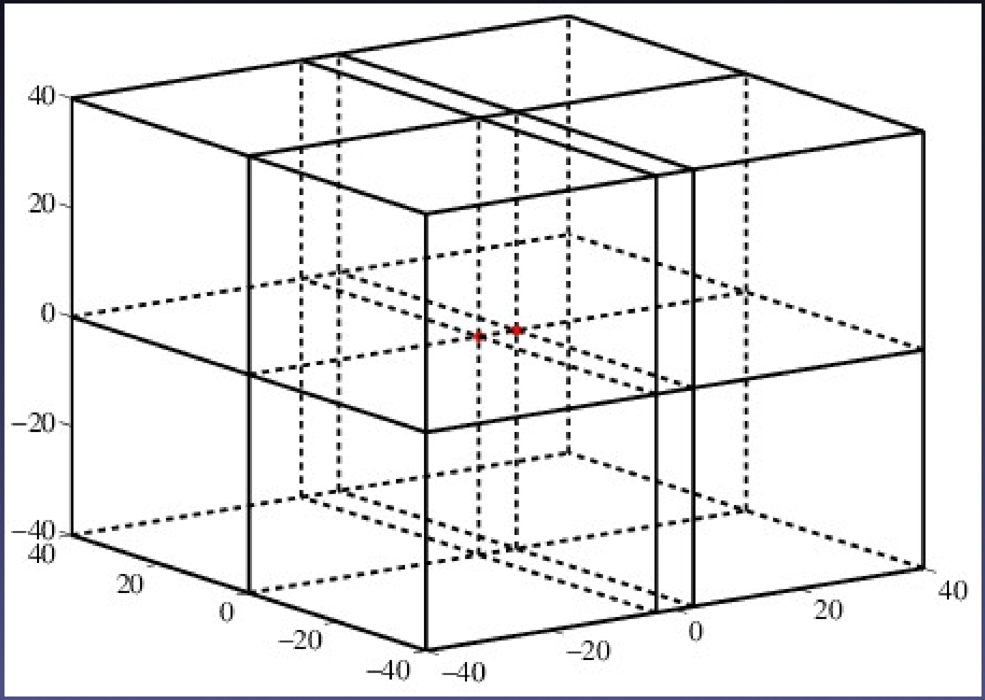




More uniform than previous decomposition

Initial mesh grid generating 3

Putting the puncture points on vertices

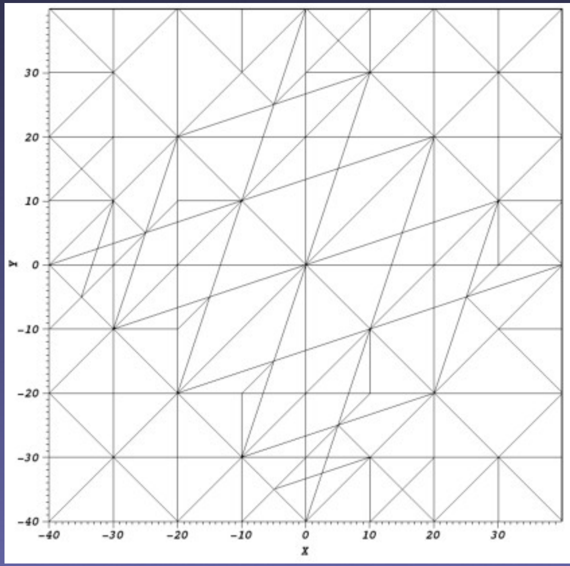


Divide the computational domain into 12 subdomains
Then use NetGen to generate mesh grid

The function is C^∞ inside of all elements

Uniform refinement

In order to avoid the pre-mature of the iteration, we need uniform refinement about 6 times and boundary refinement 3 times to resolve the whole computational domain



Adaptive refinement

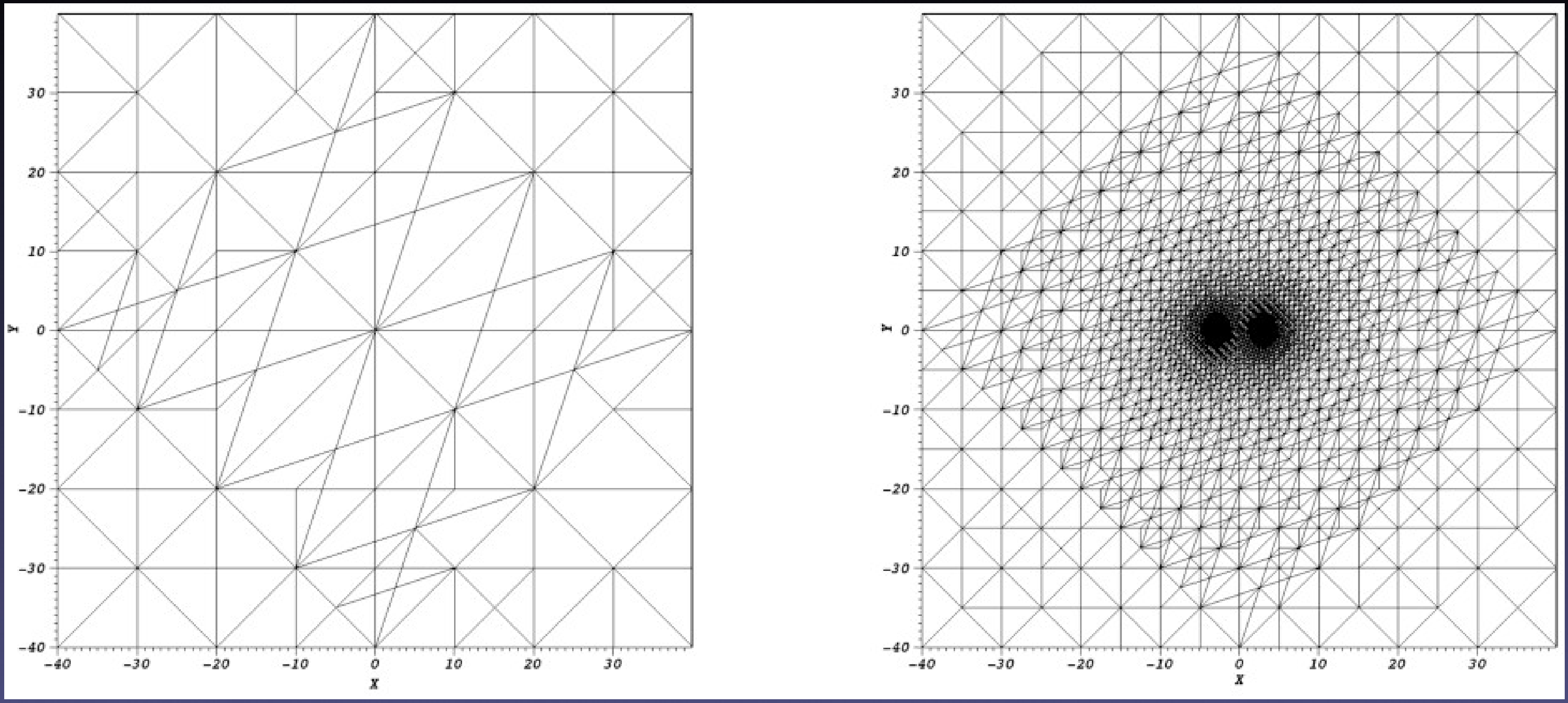
$$H \equiv \nabla^2 u_h + \frac{1}{8}\hat{K}^{ij}\hat{K}_{ij}\psi^{-7}$$

Error indicator: $$E_i \equiv \sqrt{\int_{\text{i-th element}} (H^2 + \nabla H \cdot \nabla H)\, dx^3}.$$ H_1 norm

Or : $$E_i \equiv \sqrt{\int_{\text{i-th element}} H^2 dx^3}$$ L_2 norm

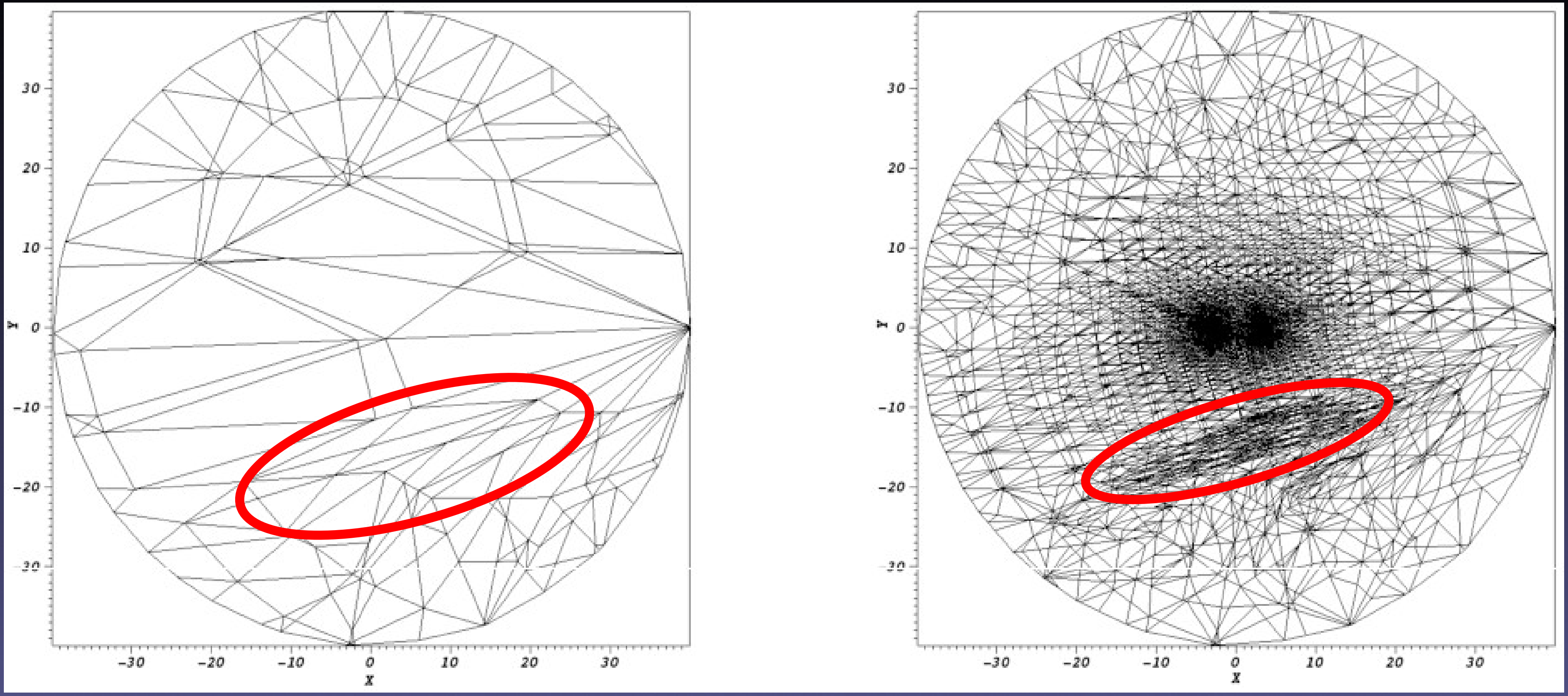
These two kind of norms result in roughly the same result in all of our tests

Adaptive refinement



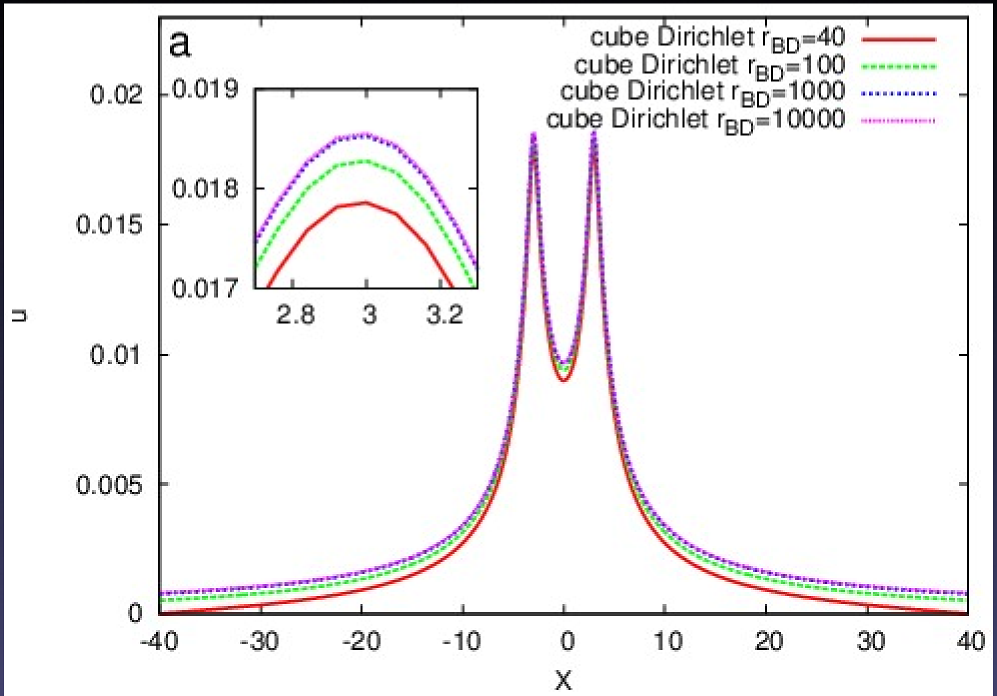
Puncture regions can be captured automatically

Boundary effect



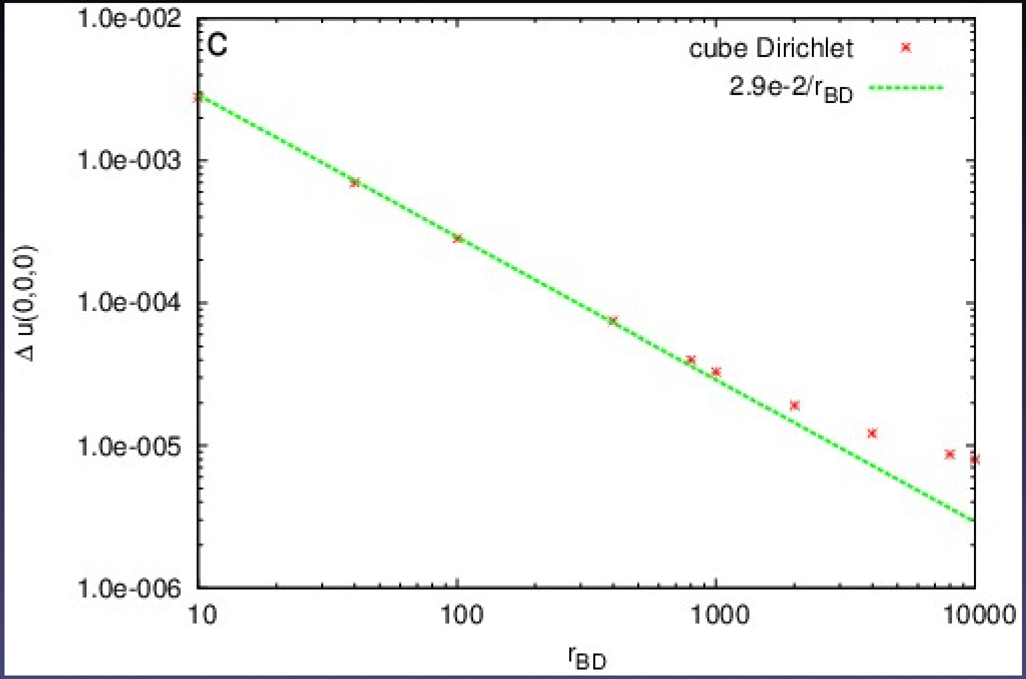
It is harder to generate efficient grid for spherical boundary than for box

Boundary effect



As expected, the boundary is farther , the numerical solutions is better

Boundary effect

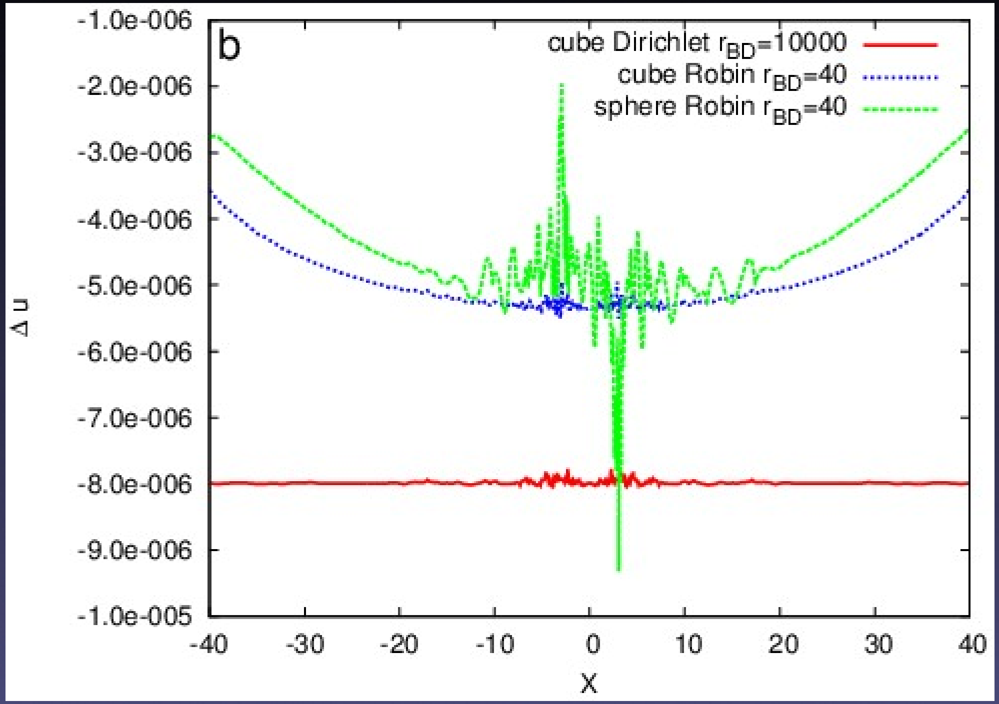


As expected, the boundary is farther , the numerical solutions is better

The convergence behaves like $\frac{1}{r_{\mathrm{BD}}}$

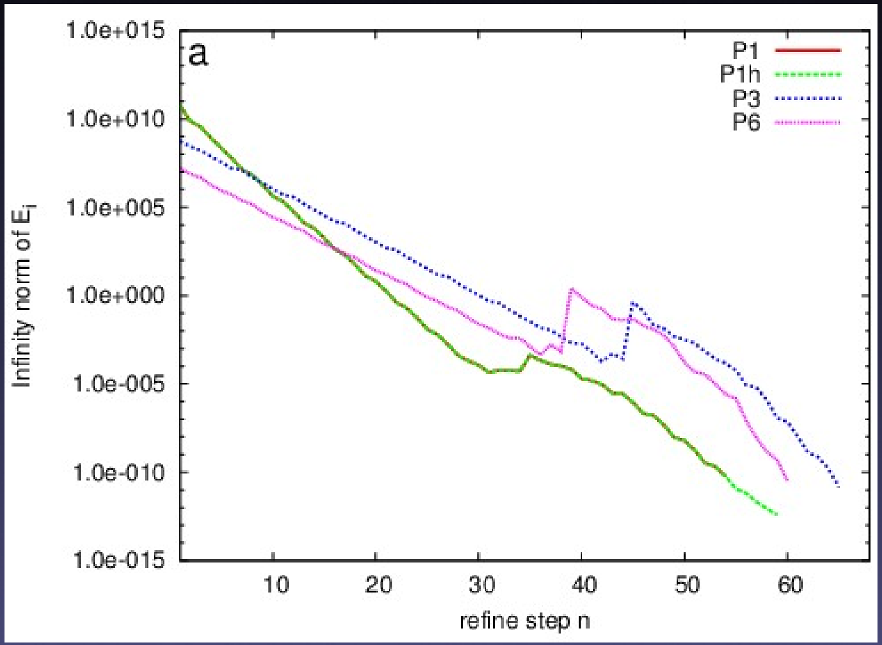
This confirms our assumption of Robin BD form

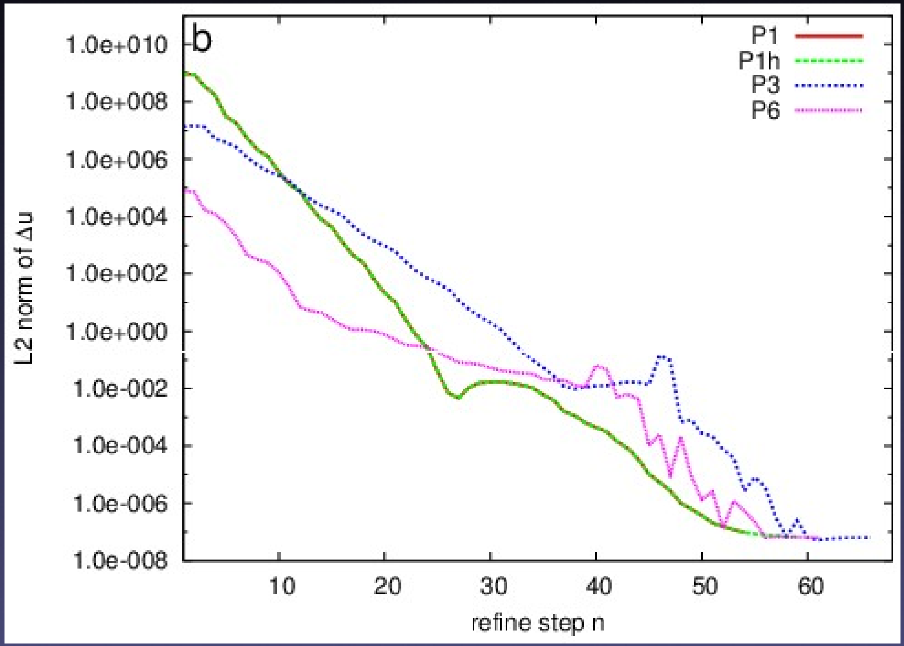
Boundary effect



The spherical configuration is less accurate than the box configuration, So the numerical error is larger than that of box

Polynomial order effect

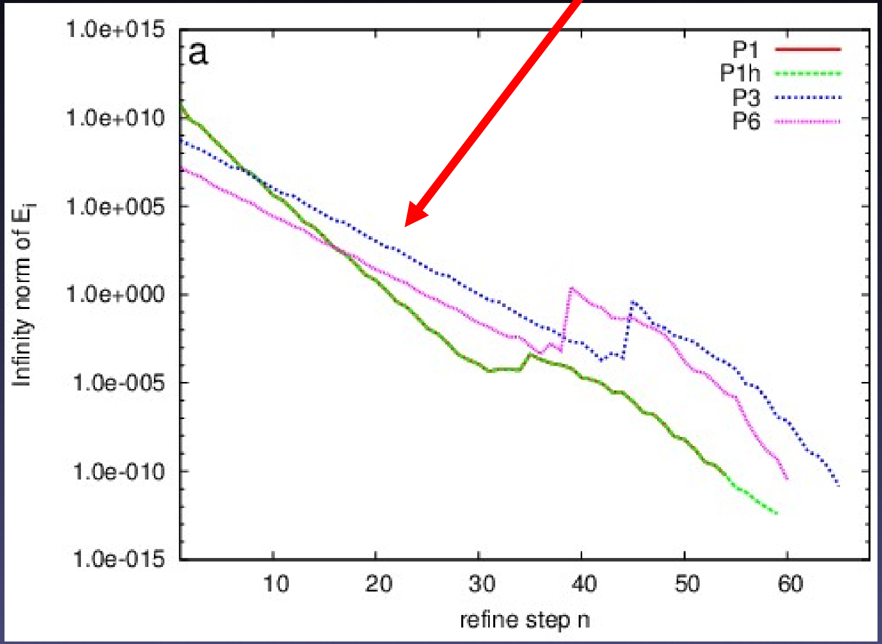


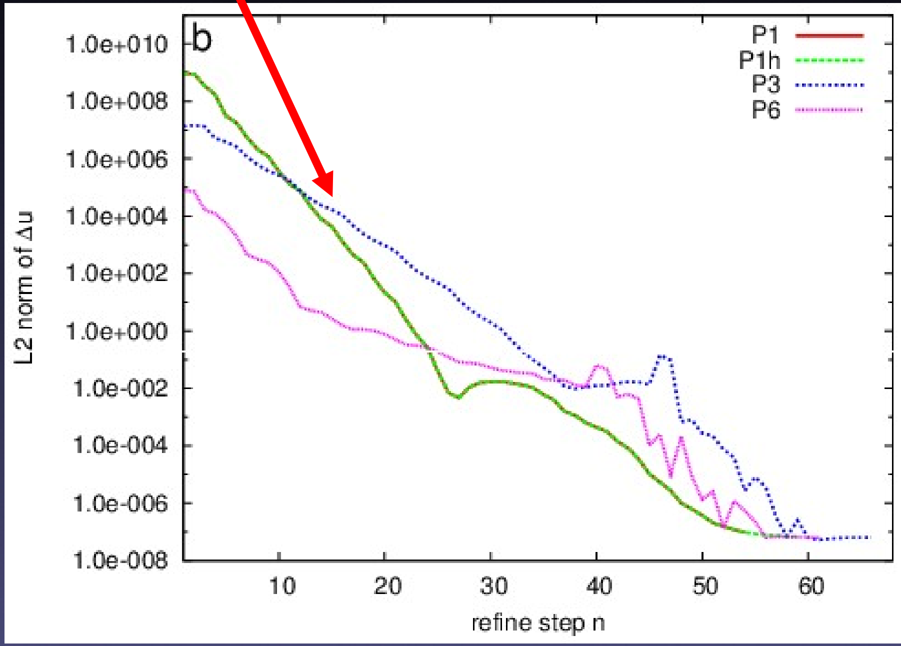


No matter we put the puncture points on vertexes or not, the results are roughly the same. These plots correspond to the result we do not put the puncture points on vertexes

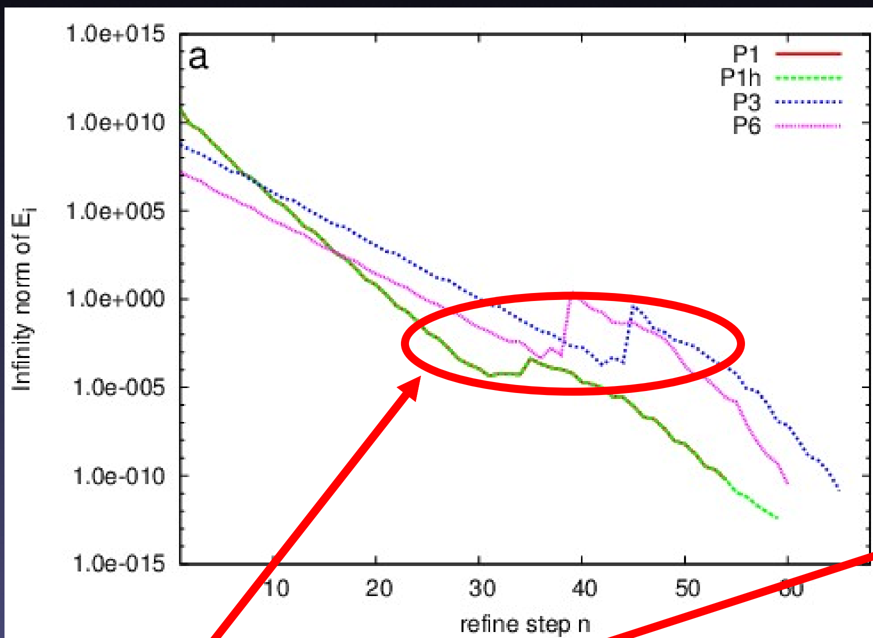
Polynomial order effect

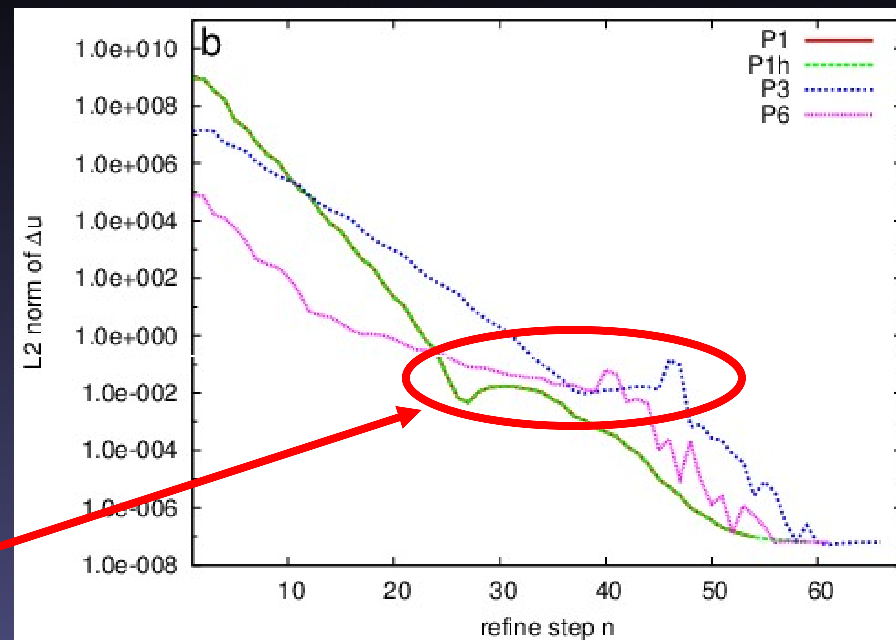
Resolve the whole domain





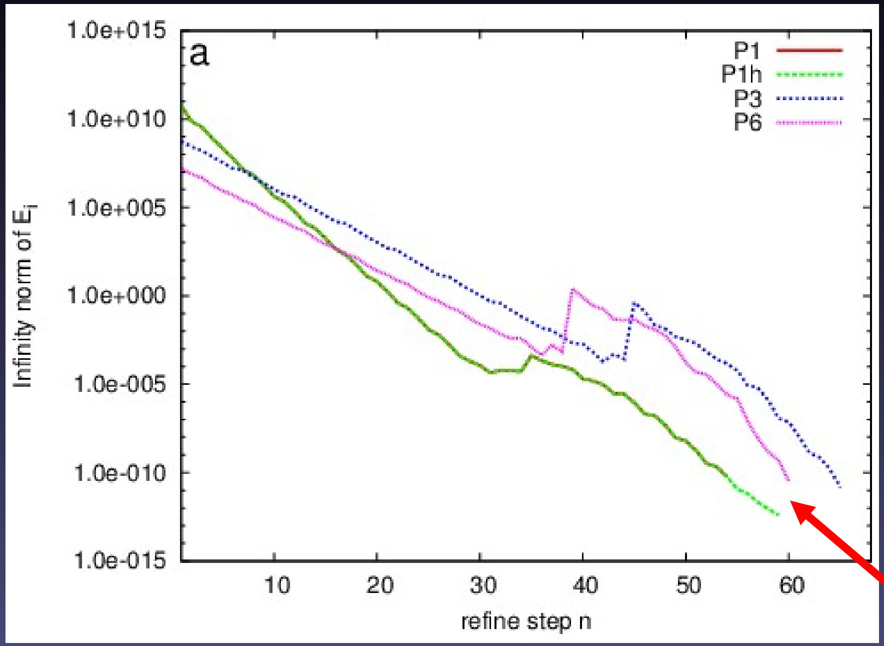
Polynomial order effect

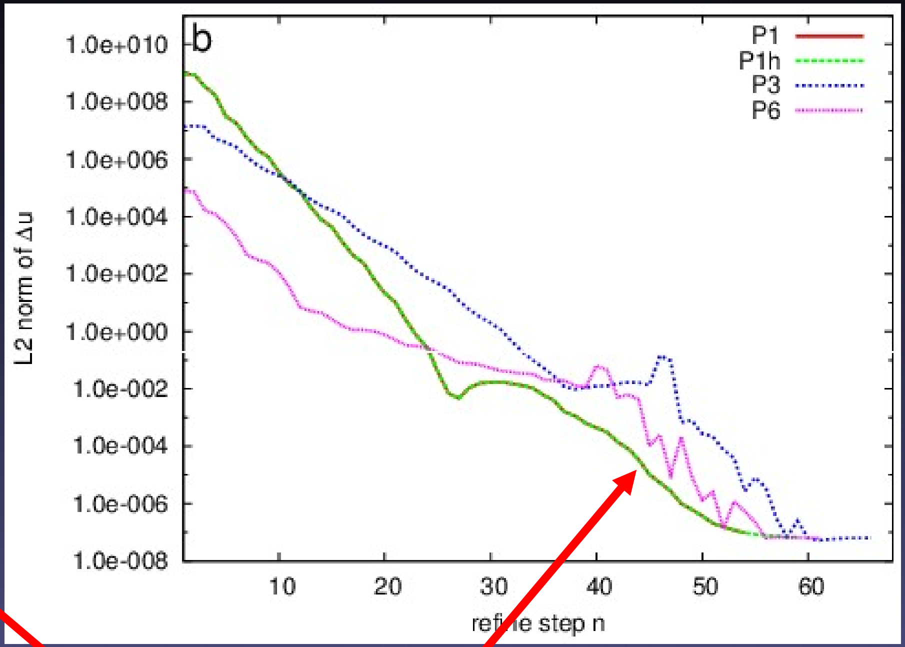




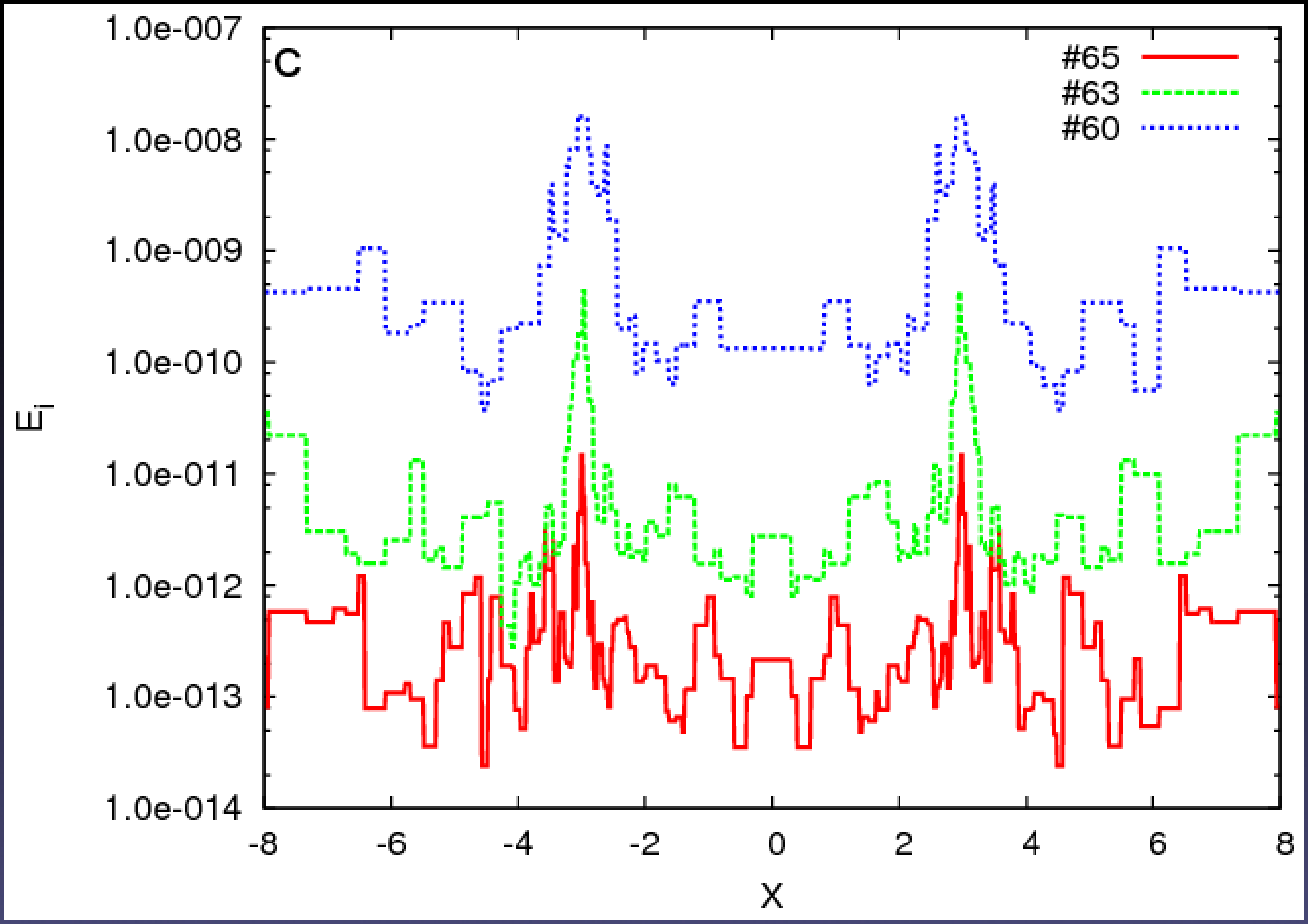
Resolve the puncture points region

Polynomial order effect



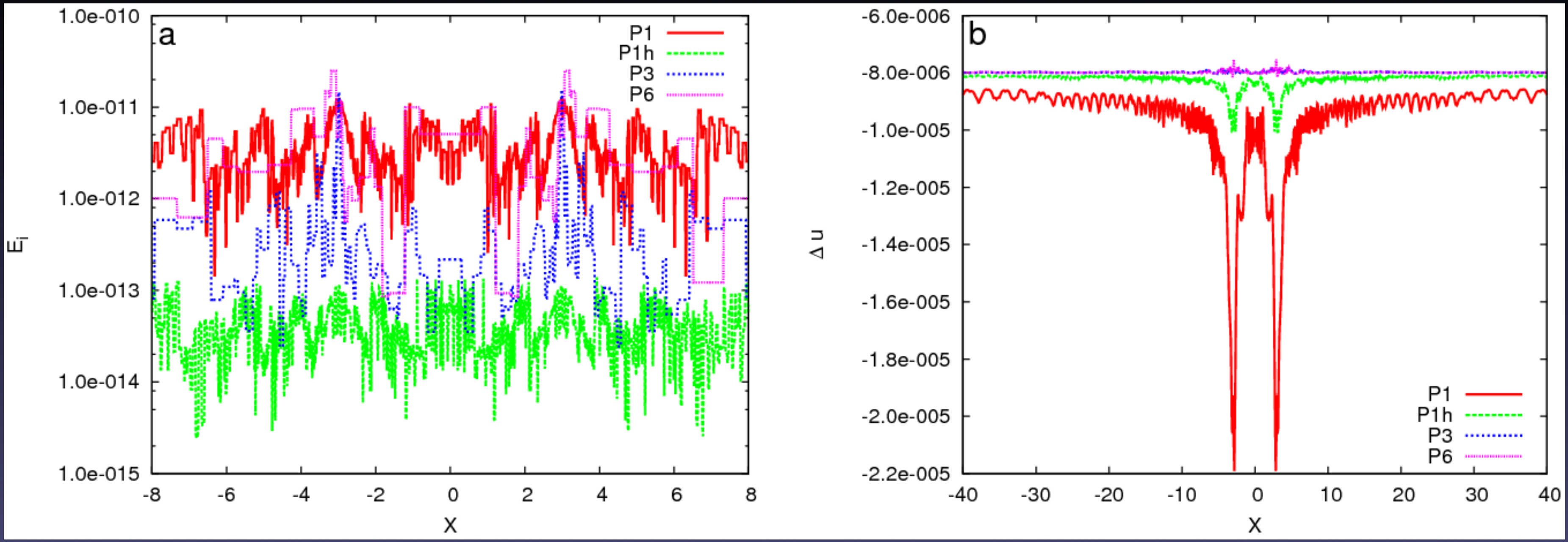


Real converge process



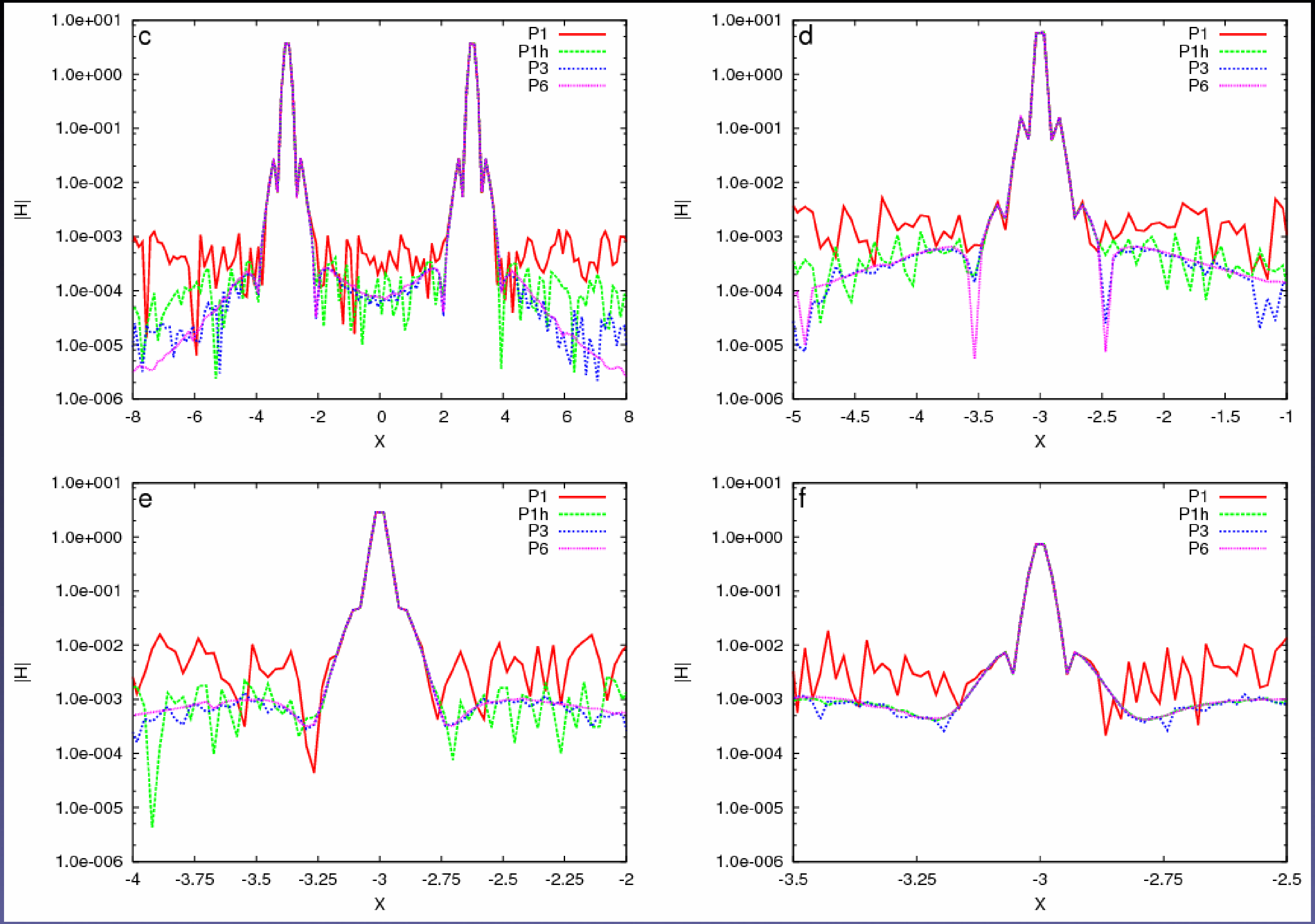
The numerical error indicator, also the Hamiltonian constraint violation. Converging behavior near puncture points

Polynomial order effect



Higher order polynomial results in more accurate solution, at the meantime the needed resolution is coarser

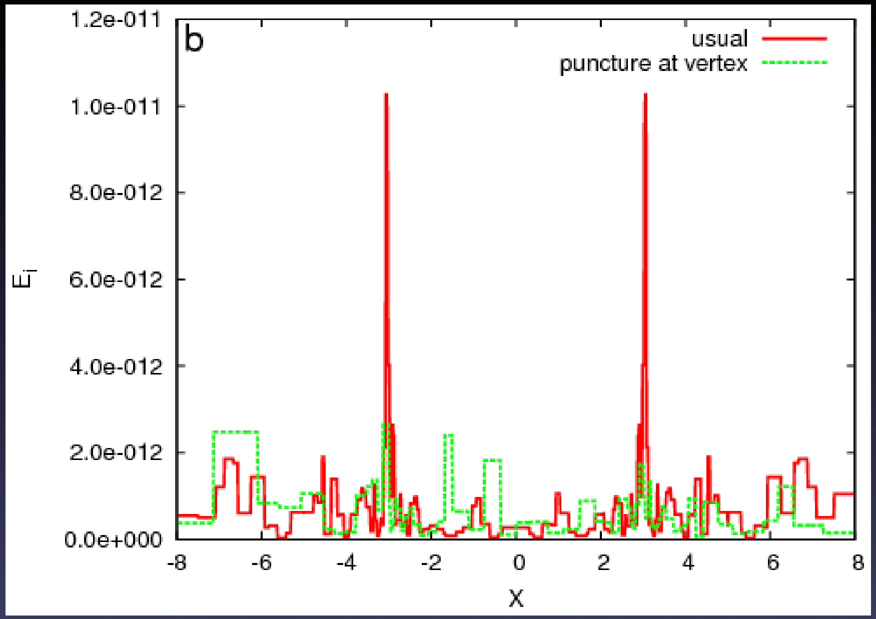
Polynomial order effect

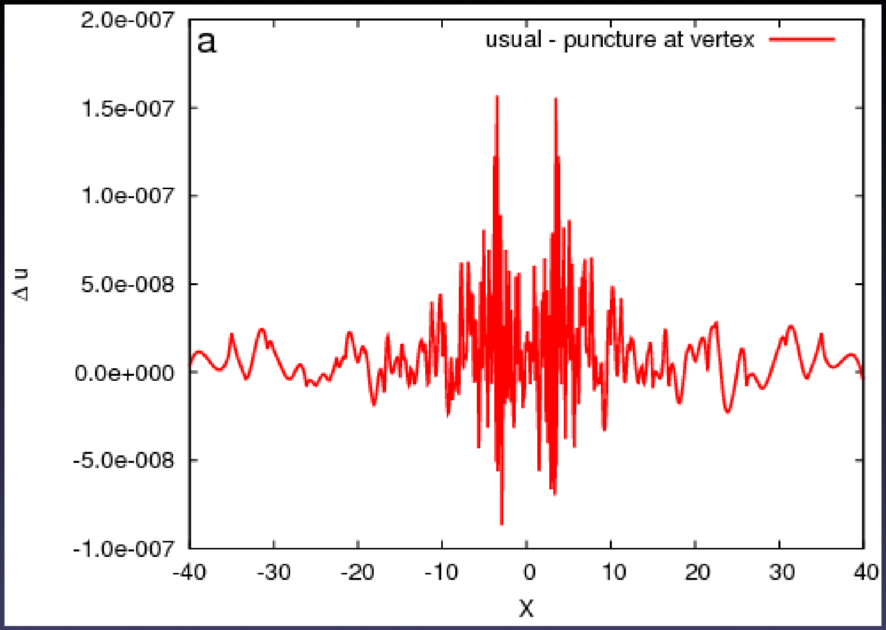


For the usage of finite difference code AMSS-NCKU.

In practice, P1 is enough

Effect of puncture point

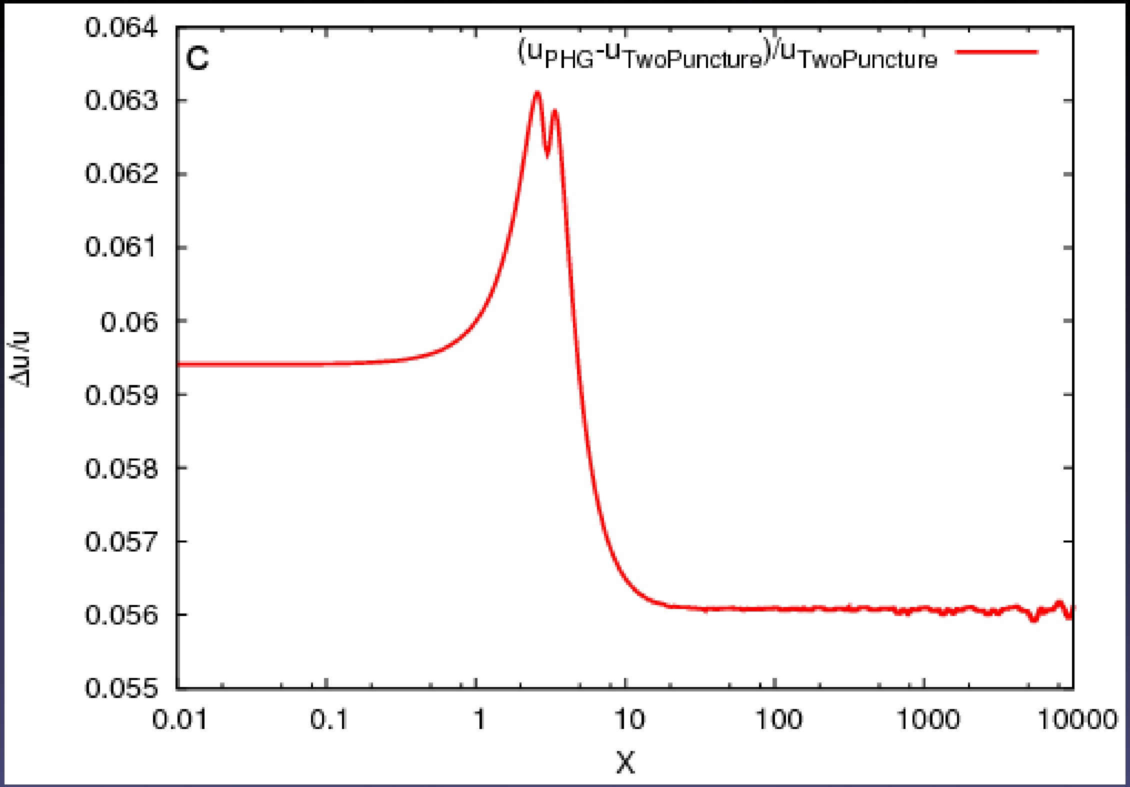




Requirement for the resolution is lower, while the constraint violation is smaller

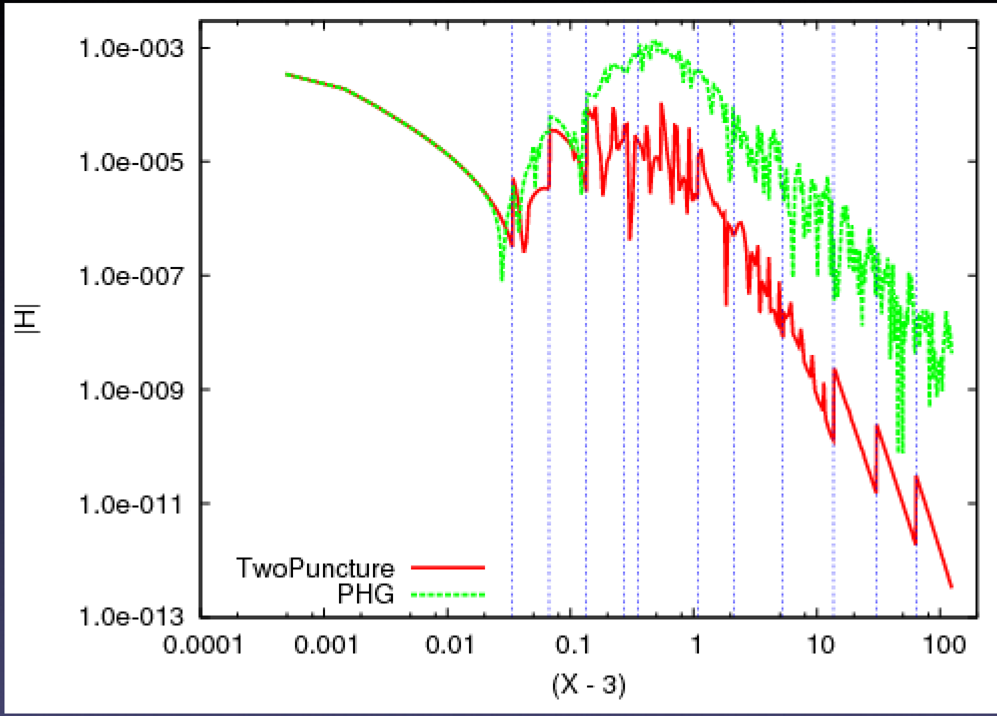
The resulted solution difference is smaller than the that of polynomial order

Comparison to spectral method



The relative difference is almost uniform among the space. If the reason is resolution, we expect the spectral method is more accurate, but AFEM is more practical and more flexible, no computational cost is wasted.

Comparison to spectral method



From practical viewpoint, we use AMSS-NCKU code to compare the solutions. Although the FEM solution does not waste resolution, spectral solution is a little bit better on constraint violation

Summary and prospect

- AFEM has been applied to solve punctured multi black hole ID
- Mesh grid and boundary conditions affect the numerical solution strongly
- High order polynomial bases and/or spectral bases can be used together with AFEM to combine the advantage of FD and spectral method. But much more work is needed
- Application to the evolution of Einstein equations is appreciated. This may provide high accuracy and high parallel efficiency

Ref: PRD 91, 044033 (2015)