Fine Coarse

$t = t_{n+2}$

$t = t_{n+1}$

$t_{n+\frac{1}{2}}$

$t = t_n$

$k_1 = \Delta t\, y_0' + O(\Delta t^{?})$ $+O(\Delta$

$k_2 = \Delta t\, y_0' + \frac{\Delta t^2}{2} y_0'' + \frac{\Delta t^3}{8}(y_0''' - f_y\, y_0'')$

$k_3 = \Delta t\, y_0' + \frac{\Delta t^2}{2} y_0'' + \frac{\Delta t^3}{8}(y_0''' - f_y\, y_0'') + O(\Delta t^{?})$

$\tilde{k}_1, \tilde{k}_2, \tilde{k}_3, \tilde{k}_4$

$= \ldots (k_3 - k_2)$ (1)

$y(t) = \sum b_i(t)\, \tilde{k}_i$

$y'(t) = \sum b_i'(t)\, \tilde{k}_i$

$y'(t) = [illegible]\, t, y)$

$P(x; A) + P(x; B) \stackrel{?}{=} P(x; A + B)$

k_1, k_2, k_3, k_4

f_n

$y_{n+1} = [\ \downarrow\ ] + y_n$

$y^{(1)} = \ldots k \cdot y^{(0)}$

$\dot{y} = f(y)$

$y^0 \sim t_n$

$y^1 \sim t_{n+\frac{1}{?}}$

$y^2 \sim t_{n+\frac{?}{?}}$

$y^3 \sim t_{n+1}$

$\dot{\phi} = \pi$

$\dot{\pi} = \phi_{xx}$

$\phi = 1$

$\pi = 1$

$\phi_0 = 1 \sim$ $\boxed{\phi \to t}$

$\pi_0 = 0$ $\boxed{\pi \simeq 1}$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = \Delta t\, f(t_n, y_1)$

$k_2 = \Delta t\, f(t_n + \frac{1}{2}\Delta t, y_2)$ $\quad y_n + \frac{1}{2} k_1$

$k_3 = \Delta t\, f(t_n + \frac{1}{2}\Delta t, y_3)$ $\quad y_n + \frac{1}{2} k_2$

$k_4 = \Delta t\, f(t_n + \Delta t, y_4)$ $\quad y_n + k_3$

1. Disable prolongation of state vector
2. Interpolate $\tilde{k}_s$ from coarser grid
3. Regular RK4 for internal points (efficient)
4. Construct $y', y'', \ldots$ with dense interpolant
5. Recompute k / f with (1)
6. Do RK4 of boundary points.
7. Reenable prolongation of state vector.