

RK4

$$y_{n+1} = y_n + \frac{1}{8}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_n$$

$$k_1 = \Delta t f(t_n, y_1)$$

$$y_2 = y_n + \frac{1}{2} k_1$$

$$k_2 = \Delta t f(t_n + \frac{1}{2} \Delta t, y_2)$$

$$y_3 = y_n + \frac{1}{2} k_2$$

$$k_3 = \Delta t f(t_n + \frac{1}{2} \Delta t, y_3)$$

$$y_4 = y_n + k_3$$

$$k_4 = \Delta t f(t_n + \Delta t, y_4)$$

$$y_{n+1} = y_n + \frac{1}{8}(k_1 + 2k_2 + 2k_3 + k_4)$$

ys in ks ("dense output formula")

$$y(t_n + \theta \Delta t) = y_n + \sum_{i=1}^4 b_i(\theta) k_i + O(\Delta t^4)$$

$$y'(t_n + \theta \Delta t) = \Delta t^{-1} \sum_{i=1}^4 k_i b_i'(\theta) + O(\Delta t^3)$$

$$y''(t_n + \theta \Delta t) = \Delta t^{-2} \sum_{i=1}^4 k_i b_i''(\theta) + O(\Delta t^2)$$

$$y'''(t_n + \theta \Delta t) = \Delta t^{-3} \sum_{i=1}^4 k_i b_i'''(\theta) + O(\Delta t)$$

$$f_y y'' = 4 \Delta t^{-3} (k_3 - k_2)$$

$$b_1'(0) = 1, b_2'(0) = b_3'(0) = b_4'(0) = 0$$

ks in ys

$$k_1 = \Delta t y'$$

$$k_2 = \Delta t y' + \frac{\Delta t^2}{2} y'' + \frac{\Delta t^3}{8} (y''' - f_y y'')$$

$$k_3 = \Delta t y' + \frac{\Delta t^2}{2} y'' + \frac{\Delta t^3}{8} (y''' + f_y y'')$$